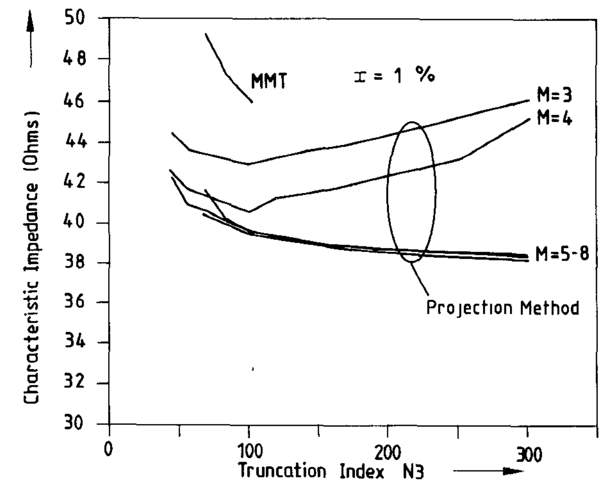

Fig. 5. The characteristic impedance of configuration 4 as a function of the truncation index $N3$ of the vector potentials and with the truncation index M of the power series as a parameter, calculated by the projection method.

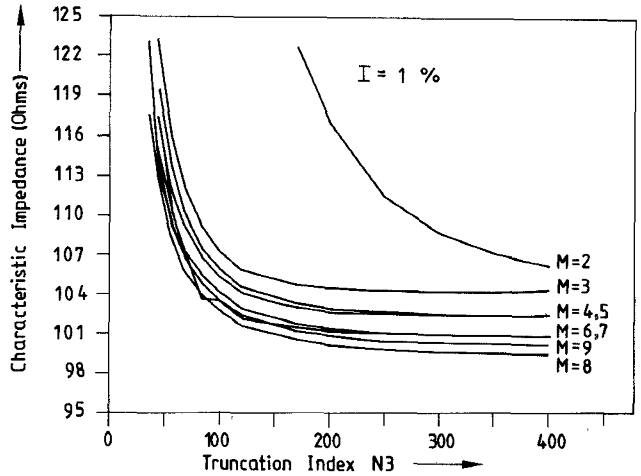

Fig. 6. The characteristic impedance of configuration 7 as a function of the truncation index $N3$ of the vector potentials and with the truncation index M of the power series as a parameter, calculated by the projection method.

error near the edge, so that the electromagnetic field (using the same truncation indices) is approximated much better by the projection method than by the MMT.

For very large metallization thicknesses (configurations 5 and 6: $t = 1$ mm, nearly twice the substrate height), both the projection method and the MMT fail. Considering a more realistic configuration, for example, a quadratic metal strip on a GaAs substrate of the type used in monolithic microstrip integrated circuits (MMIC) (configuration 7: $\epsilon_r = 12.8, 2e = 600$ μm, $c = 400$ μm, $t = w = 2d = 5$ μm, $h = a = 100$ μm, $f = 10$ GHz, $(k_z/k_0)^2 = 6.4$), a good convergence behavior is obtained, whereas the conventional MMT delivers no results again within the ordinate range (Fig. 6). Using the projection method, numerical problems occur for a truncation index $M = 10$.

IV. Conclusions

The application of a projection method, based on an idea by Jansen [1], delivers good results for the characteristic impedance of microstrip configurations with finite metallization thickness. It should be emphasized that the procedure of the projection method is a general one which can be applied to various boundary value problems.

Acknowledgment

The diligent support of Mr. S. Koßlowski and many helpful discussions with Dr. P. Waldow are thankfully acknowledged.

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Parametric Equations for Surface Waves in Dielectric Slab

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***Abstract*—For the dielectric slab it is shown that 1) the dispersion curve for the nth surface wave can be found using parametric equations in which the normalized inside wavenumber K_{x1} and the mode number are the parameters, 2) the dispersion curve for the nth surface wave mode can also be found by using parametric equations in which the mode number and a modified wavenumber x' with common domain $[0, \pi/2]$ are the parameters, and 3) all TE or all TM dispersion curves for surface waves are related to each other by a simple algebraic equation using the mode numbers and the normalized propagation constants K_0 and β as the variables.**

I. Fundamental Equations

Presently, dispersion curves for surface waves in dielectric slab are obtained using either a graphical or a computer technique [1], [2]. These techniques are unnecessary since the dispersion curves can be obtained much more easily using parametric equations. In addition, these graphical or computer techniques obscure the simple algebraic relation between two different TE or two different TM surface waves. This simple algebraic relation can be used to express the mth surface wave in terms of the nth surface wave propagation constants.

The normalized dispersion equations (normalized w.r.t. the slab width $2d$) for surface waves in dielectric slab are

$$K_{x1} \tan K_{x1} = \delta K_{x2} \qquad \text{(symmetric modes)} \qquad (1a)$$

$$K_{x1} \operatorname{ctn} K_{x1} = -\delta K_{x2} \qquad \text{(antisymmetric modes)} \qquad (1b)$$

where $\delta = 1$ for TE modes, or $\delta = \epsilon_r/\epsilon_0$ (the ratio of the relative permittivities) for TM modes [1]–[3]. Since the value of K_{x2} must be positive, K_{x1} lies in the range

$$n\frac{\pi}{2} \leq K_{x1} \leq (n+1)\frac{\pi}{2}, \quad n = 0, 1, 2, \cdots \qquad (2)$$

Manuscript received January 23, 1987; revised June 1, 1987.

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IEEE Log Number 8716174.

0018-9480/87/1000-0921$01.00 ©1987 IEEE

where the mode number n is even for symmetric modes and odd for antisymmetric modes. The normalized wavenumbers K_{x1} and K_{x2} are related to the normalized propagation constants β and K_0 through the following:

$$K_{x1}^2 = \kappa K_0^2 - \beta^2 \tag{3a}$$

$$K_{x2}^2 = \beta^2 - K_0^2 \tag{3b}$$

where κ is the ratio of the relative permittivities, ϵ_r/ϵ_0. Note that the normalized variables K_{x1}, K_{x2}, β, and K_0 equal their unnormalized value multiplied by half the slab width.

II. Formulations

Rearranging (3) yields

$$K_0^2 = \frac{K_{x1}^2}{(\kappa - 1)}\left[1 + \frac{K_{x2}^2}{K_{x1}^2}\right] \tag{4a}$$

$$\beta^2 = \frac{K_{x1}^2}{(\kappa - 1)}\left[1 + \kappa\frac{K_{x2}^2}{K_{x1}^2}\right]. \tag{4b}$$

Using standard reduction formulas for the tangent and cotangent functions, (1a) and (1b) are combined into the following single equation:

$$\tan x' = \left\{\begin{array}{c} \tan\left(x' + n\frac{\pi}{2}\right) \\ -\operatorname{ctn}\left(x' + n\frac{\pi}{2}\right)\end{array}\right\} = \delta\frac{K_{x2}}{K_{x1}}. \tag{5}$$

Here the range of x' is $0 \leqslant x' \leqslant \pi/2$, $\delta = 1$ for TE modes or $\delta = \epsilon_r/\epsilon_0$ for TM modes, and $n = 0, 2, 4, \cdots$ for symmetric modes (upper equation in (5)) while $n = 1, 3, 5, \cdots$ for antisymmetric modes (lower equation in (5)). Note that the wavenumber K_{x1} equals $x' + n(\pi/2)$. Substituting (5) into (4a) and (4b) results in

$$K_0^2 = \frac{\left(x' + n\frac{\pi}{2}\right)^2\left(1 + \frac{\tan^2 x'}{\delta^2}\right)}{(\kappa - 1)} \tag{6a}$$

$$\beta^2 = \frac{\left(x' + n\frac{\pi}{2}\right)^2\left(1 + \frac{\kappa\tan^2 x'}{\delta^2}\right)}{(\kappa - 1)}. \tag{6b}$$

Equations (6a) and (6b) are parametric equations where the modified wavenumber x' is the parameter with a common domain $[0, \pi/2]$. These equations can easily be used to plot any dispersion curve for the nth TE or TM mode of dielectric slab. A curve is determined for a particular mode by picking an appropriate value for the mode number n and varying the value of x' from zero to $\pi/2$.

Dividing (6b) by (6a) and taking the square root results in

$$\frac{\beta}{K_0} = \left[\frac{\delta^2 + \kappa\tan^2 x'}{\delta^2 + \tan^2 x'}\right]^{1/2}. \tag{7}$$

Equation (7) represents the normalized propagation constant or the effective guided mode refractive index. This equation is a function of x' and is independent of the mode number n. Clearly, a particular value of x' results in the same normalized propagation constant for all TE modes or for all TM modes. This result can be used to determine the propagation constants needed to produce a desired effective guided mode index. Equation (7) would be solved for x', and this value of x' would then be used in (6) along with the appropriate mode number n.

Equation (6) can also be used to show that the dispersion curves for two different TE modes or two different TM modes are related by a simple algebraic equation. Let K_{0_n} and β_n represent the known propagation constants for the nth TE or TM mode, and let K_{0_m} and β_m be the desired propagation constants for the mth TE or TM mode. If x' is a constant, then (6) yields

$$\frac{K_{0_n}}{K_{0_m}} = \left[\frac{x' + n\frac{\pi}{2}}{x' + m\frac{\pi}{2}}\right] = \frac{\beta_n}{\beta_m}. \tag{8}$$

Rearranging (8) with the aid of (3) yields

$$K_{0_m} = \frac{\left(\kappa K_{0_n}^2 - \beta_n^2\right)^{1/2} + (m - n)\frac{\pi}{2}}{\left(\kappa K_{0_n}^2 - \beta_n^2\right)^{1/2}} K_{0_n} \tag{9a}$$

$$\beta_m = \frac{\left(\kappa K_{0_n}^2 - \beta_n^2\right)^{1/2} + (m - n)\frac{\pi}{2}}{\left(\kappa K_{0_n}^2 - \beta_n^2\right)^{1/2}} \beta_n. \tag{9b}$$

This simple algebraic equation relates all TE or all TM dispersion curves using only the mode numbers and the propagation constants. Therefore, given the dispersion curve for the nth mode, the dispersion curve for the mth mode can be obtained by employing (9) rather than resolving the dispersion equations (1a) and (1b). It is also observed that the relation between β_m and β_n is the same as the relation between K_{0_m} and K_{0_n}.

III. Conclusions

Parametric equations have been developed to determine the dispersion curves of surface waves in dielectric slab. This method is more convenient to apply than either a graphical or a computer technique. In addition, it has been shown that any two TE modes or any two TM modes are related by a simple algebraic equation.

Acknowledgment

The author wishes to express his appreciation to Prof. H. Zmuda of Stevens Institute of Technology for his helpful discussion.

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